

Peculiarities of the resistive transition in fractal superconducting structures

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The influence of fractal clusters of a normal phase on the current-voltage characteristics of a percolation superconductor in the region of a resistive transition has been studied. The clusters represent the aggregates of columnar defects, which give rise to a correlated microscopic disorder in the system. Dependencies of the static and dynamic resistance on the transport current are obtained for an arbitrary fractal dimension of the cluster boundaries. It is revealed that a mixed state of the vortex glass type is realized in the superconducting system involved.

Fractal superconducting structures possess a number of unusual magnetic and transport properties [1] - [3]. The possibility to enhance pinning by trapping the magnetic flux in the normal phase clusters with fractal boundaries [4]-[6] is of much interest. The study of peculiarities of the current-voltage (V - I) characteristics in the region of a resistive transition allows getting new information about the nature of the vortex state in such systems.

The problem setting was described in detail in Refs. [1], [2]. We consider the superconductor containing columnar fragments of a normal phase that represents either inclusions of different chemical composition or regions with reduced superconducting order parameter. Such columnar defects can be formed in the course of the film growth process or induced by heavy ion bombardment [7], [8]. The columnar defects can produce a much more intensive pinning than, for example, the point defects, because their topology is closer to a vortex structure [9], [10]. The presence of columnar defects in a superconductor enhances irreversible magnetization and, on the other hand, suppresses the magnetic flux creep, which allows the critical current to be increased up to a value of depairing current [11], [12].

When a sample is cooled in a magnetic field below the critical temperature, the magnetic flux is trapped in isolated clusters of a normal phase. Then a transport current is passed through the sample transversally to the magnetic field. The transport current is added to all the persistent superconducting currents, which circulate around the normal phase clusters and maintain distribution of the trapped magnetic flux to be unchanged. By a cluster we imply a set of columnar defects united by a common trapped flux and surrounded by the superconducting phase. Since the distribution of the trapped magnetic flux is two-dimensional, we will consider the transverse cross-section of the clusters, representing extended objects, by a plane carrying the transport current. As it was found for the first time in Ref. [1], the normal phase clusters can have fractal boundaries. This feature essentially affects the dynamics of a trapped magnetic flux [2], [5], [6]. In the following, we will consider a special case when characteristic sizes of the normal phase clusters much exceed both the coherence length and the penetration depth. This assumption well agrees with the data about the cluster structure of YBCO high-temperature superconducting films [1], [2] and is most consistent with the important role of cluster boundaries in magnetic flux trapping.

It is assumed that a superconducting percolation cluster is formed in the plane where the electric current flows. Such a structure provides for an effective pinning, since the magnetic field is trapped in finite clusters of the normal phase. As the transport current increases, a moment comes when vortices begin to break away from the clusters of weaker pinning force than the Lorentz force created by the current. With gradually increasing transport current, the vortices will break away first from the clusters of a smaller pinning force and, therefore, of a lower critical current. The vortices will travel through the superconducting space via weak links, which connect the clusters of the normal phase between themselves and act as the channels for magnetic flux transport. These weak links are formed especially readily at various structural defects in high-temperature superconductors that have small coherence length. Various structural defects, which would cause an additional scattering at long coherence length, give rise to the weak links in

high-temperature superconductors.

The motion of vortices broken away from pinning centers results in a voltage drop across the sample that leads to the transition into a resistive state. Each cluster has an individual configuration of weak links so it gives the contribution to the total statistical distribution of critical currents. The critical current of a cluster is proportional to the pinning force and is equal to the current at which the magnetic flux ceases to be held inside the normal phase cluster. Different types of the critical current distribution for the clusters with fractal boundaries were considered in [3], [4], [6], [13].

The further consideration is restricted to the most practically important case of an exponential-hyperbolic distribution of critical currents:

$$f(i) = \frac{2C}{D} i^{-2/D-1} \exp(-C i^{-2/D}), \quad (1)$$

which is realized in the YBCO based film structures with exponential distribution of the cluster areas [1], [2]. In the expression of Eq. (1) $i \equiv I/I_c$ is the dimensionless current normalized to the critical current of the transition into the resistive state, $I_c = \alpha (C\bar{A})^{-D/2}$ is the transport current, D is the fractal dimension of the cluster boundary, $C \equiv ((2+D)/2)^{2/D+1}$ is the constant depending on the fractal dimension, \bar{A} is the average cluster area, and α is the cluster form factor.

The fractal dimension D sets a scaling relationship between the perimeter P and area A of the normal phase cluster: $P^{1/D} \propto A^{1/2}$. This relation is consistent with the generalized Euclid theorem, according to which the ratios of corresponding geometric measures are equal when reduced to the same dimension [14]. For the Euclidean clusters, the fractal dimension coincides with the topological dimension of line ($D = 1$), while the dimension of fractal clusters always exceeds their topological dimension ($D > 1$) to reach maximum ($D = 2$) for the clusters of the most fractality. A fractal object has fractional dimension that reflects a highly indented shape of its boundary [14]. Let us note that the probability density for the exponential-hyperbolic distribution of Eq. (1) is equal to zero at $i = 0$, which implies the absence of any contribution from negative and zero currents. That will allow us to avoid any artificial assumption about the existence of a vortex liquid, which has a finite resistance in the absence of transport currents because of free vortices presence: $r(i \rightarrow 0) \neq 0$. Such an assumption is made, for example, in the case of the normal distribution of critical currents [15].

The voltage drop across a superconductor in the resistive state represents an integral response of all clusters to the action of transport current:

$$u = r_f \int_0^i (i - i') f(i') di' \quad (2)$$

where u is the dimensionless voltage and r_f is the dimensionless resistance of the flux flow. Using the convolution integral of type (2), it is possible to find the V - I characteristics of fractal superconducting structures for an arbitrary fractal dimension as well as to study the dependence of resistance on the transport current. The resistive characteristics provide important information about the nature of the vortex state. The standard parameters in this case are the dc (static) resistance $r \equiv u/i$, and the differential (dynamic) resistance $r_d \equiv du/di$. The corresponding dimensional quantities R and R_d can be found using the formulas $R = rR_f/r_f$ and $R_d = r_dR_f/r_f$, where R_f is the dimensional flux flow resistance.

For an exponential-hyperbolic distribution of critical currents described by Equation (1), expressions for the resistances of a superconductor with fractal clusters of the normal phase are as follows:

$$r = r_f \left(\exp(-C i^{-2/D}) - \frac{C^{D/2}}{i} \Gamma\left(1 - \frac{D}{2}, C i^{-2/D}\right) \right) \quad (3)$$

$$r_d = r_f \exp(-C i^{-2/D}) \quad (4)$$

where $\Gamma(\nu, z)$ is the complementary incomplete gamma-function. In the limiting cases of the Euclidean clusters ($D = 1$) and the clusters of most fractal boundaries ($D = 2$), the above formulas can be simplified:

$D = 1$:

$$r = r_f \left(\exp \left(-\frac{3.375}{i^2} \right) - \frac{\sqrt{3.375\pi}}{i} \operatorname{erfc} \left(\frac{\sqrt{3.375}}{i} \right) \right)$$

$$r_d = r_f \exp \left(-\frac{3.375}{i^2} \right)$$

where $\operatorname{erfc}(z)$ is the complementary error function, and

$D = 2$:

$$r = r_f \left(\exp \left(-\frac{4}{i} \right) + \frac{4}{i} \operatorname{Ei} \left(-\frac{4}{i} \right) \right)$$

$$r_d = r_f \exp \left(-\frac{4}{i} \right)$$

where $\operatorname{Ei}(z)$ is the exponential integral function.

Figure 1 shows the graphs of the dc resistance as a function of transport current for superconductor with fractal clusters of a normal phase. The curves drawn for the Euclidean clusters ($D = 1$) and for the clusters of the most fractal boundaries ($D = 2$) bound a region containing all the resistive characteristics for an arbitrary fractal dimension. As an example, the dashed curve shows the case of the fractal dimension $D = 1.44$ found from the data of the geometric probability analysis of the electron micrographs of YBCO superconducting film structures [1]. The dependencies of resistance on the current shown in Fig. 1 are typical of the vortex glass, whereby the curves plotted in a double logarithmic scale are convex and the resistance tends to zero as the transport current decreases, $r(i \rightarrow 0) \rightarrow 0$ which is related to suppression of the magnetic field creep [15], [16]. A vortex glass represents an ordered system of vortices, which has no long-range ordering. At the same time, the vortex configuration is stable in time and can be characterized by the order parameter of the glassy state [17], [18]. In the H - T phase diagram, mixed state of the vortex glass type exists in the region below the irreversibility line. The dashed straight line at the upper right of Fig. 1 corresponds to a viscous flux flow regime ($r = r_f = \text{const}$), which can be achieved asymptotically only. Since the V - I characteristic of Eq. (2) of a fractal superconducting structure is nonlinear, the dc resistance (3) is not constant and depends on the transport current. In this situation, important information can be provided by the differential resistance, a small-signal parameter determined by the slope of the V - I characteristic. The plots of the differential resistance versus transport current are qualitatively analogous to the resistance curves presented in Fig. 1. A difference between these characteristics can be seen in Fig. 2. As the fractal dimension grows, the two parameters behave more like each other, but in the latter case there is a difference in a wider range of transport currents. This is related to the fact that, as the fractal dimension increases, the exponential-hyperbolic distribution of critical currents of Eq. (1) broadens out moving toward greater i values [2], [13].

The differential resistance is determined by the density of vortices broken away from pinning centers by the transport current i ,

$$n(i) = \frac{B}{\Phi_0} \int_0^i f(i') di' = \frac{B}{\Phi_0} \exp \left(-C i^{-2/D} \right) \quad (5)$$

where B is the magnetic field, $\Phi_0 \equiv hc/(2e)$ is the magnetic flux quantum, h is the Planck constant, c is the speed of light, and e is the electron charge. A comparison of expressions of Eqs. (4) and (5) shows that the differential

resistance is proportional to the density of free vortices: $r_d = (r_f \Phi_0 / B)n$. The resistance of a superconductor in the resistive state is determined by the motion just of these vortices because it gives rise to the voltage across the sample.

The vortices are broken away from pinning centers mostly when $i > 1$, that is, above the resistive transition. A practically important feature of the fractal superconducting structures consists in that the fractal character of cluster boundaries enhances pinning [5] and, hence, decreases the electric field induced in the superconductor by the magnetic flux motion [4]. This is demonstrated both in Fig. 1, where the resistance decreases with increasing fractal dimension above the resistive transition, and in Fig. 2, where an increase in the fractal dimension of cluster boundaries leads to a decrease in the relative difference between the differential resistance and the dc resistance.

In the range of transport currents below the resistive transition ($i < 1$), the situation changes to opposite: the resistance increases for the clusters of greater fractal dimension (Fig. 1). This behavior is related to the fact that, as the fractal dimension increases, the exponential-hyperbolic distribution of critical currents of Eq. (1) broadens out covering both high and small currents. For this reason, the breaking of the vortices away under the action of transport current begins earlier for the clusters of greater fractal dimension. The range of currents involved is characterized by a small number of free vortices (which is much smaller than above the resistive transition) and, accordingly, by a low resistance (Fig. 1). This interval corresponds to the so-called initial fractal dissipation region, which was observed in BPSCCO samples with silver inclusions as well as in polycrystalline YBCO and GdBCO samples [19].

Thus, the fractal properties of the clusters of a normal phase significantly influence the resistive transition. This phenomenon is related to the properties of the fractal distribution of critical currents. The relations established between the resistance and the transport current correspond to a mixed state of the vortex glass type. An important result is that the fractal character of the normal phase clusters enhances pinning, thus decreasing the resistance of a superconductor above the transition into a resistive state.

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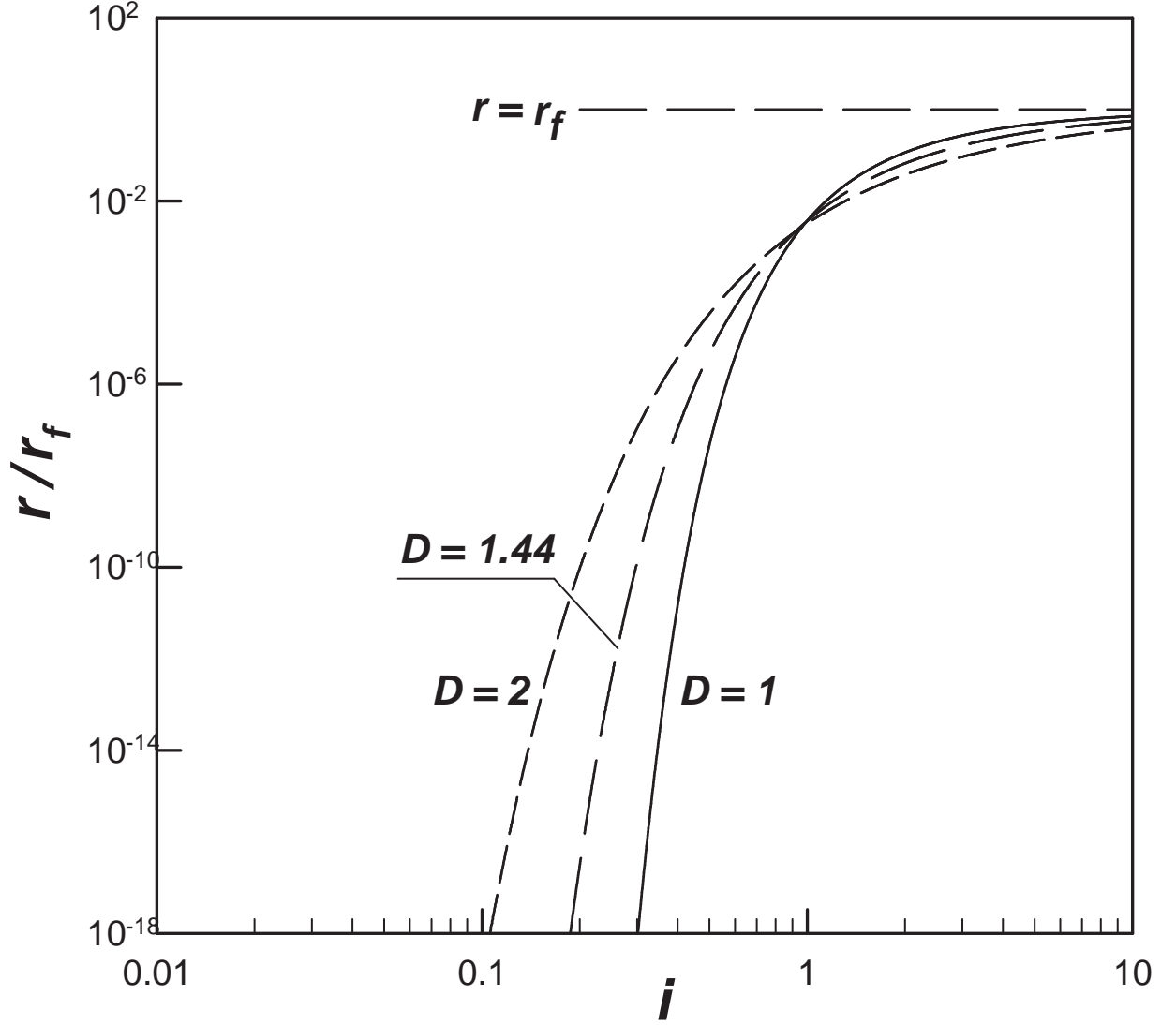


FIG. 1. Dependence of the *dc* resistance on the transport current for superconductor with fractal normal phase clusters (D is the fractal dimension). The dashed horizontal line $r = r_f$ at the upper right corresponds to the viscous flow of a magnetic flux.

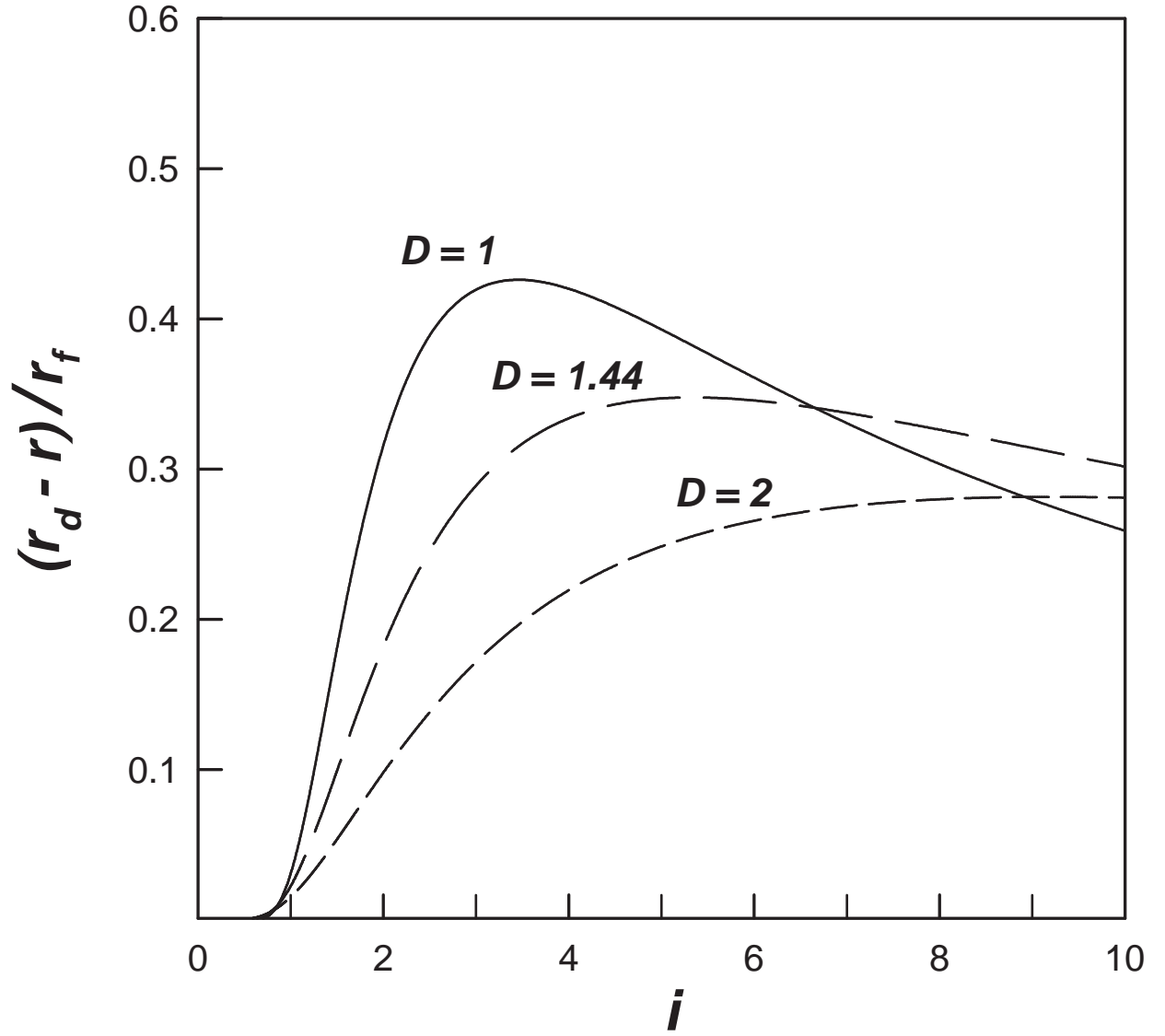


FIG. 2. A comparison of the *dc* resistance and the differential resistance for a superconductor with fractal normal phase clusters of various dimensions.